DETERMINING THE MOVEMENT OF A LIQUID FOR A CONDITION SPECIFIED ON A STREAMLINE

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16. Abstract

A treatment of some problems concerning liquid streamlining. Mathematical treatment of selected problems giving optimum solutions for free jet canals and for determining the width and length of such a canal under specified conditions. Consideration of the possible application of this method to other similar problems.

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§ 1. In our study, reported at the VIII Congress of Russian Natural /89*
Scientists and Physicians[1], we showed how concepts concerning generating and guidance grids can be employed for the primary solutions of problems in determining the flow of a liquid at a constant velocity, specified on an unknown streamline[2]. In this regard, we have in mind the application of the aforementioned method of investigation to those cases in which a certain relationship between velocity v and angle θ of its inclination to axis ox must be satisfied on a boundary line.

§ 2. In all investigations of free jets it is assumed that no forces are acting on the liquid; now we shall examine a liquid under the influence of gravity. Let the direction of axis ox coincide with the direction of the force of gravity. The velocity of the liquid on the edge of the jet is expressed by the formula:

$$v^2 = 2gx + \text{const.}, \tag{1}$$

where g is the pull of gravity. We take the differential from both parts of /90
this formula, passing along the edge of the jet in the direction of the flow of liquid to arc element ds. We obtain:

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[1] "The Modified Kirchoff Method for Determining the Flow of a Liquid in Two Dimensions, and at a Constant Velocity Specified on an Unknown Streamline," *Matematicheskiy Sbornik'* [Mathematics Textbook], Vol. XV, 1890.

[2] Almost simultaneously with our report, Michell read a report at the Royal Society in London entitled, "On the Theory of Free Streamlines" (Philosophical Transactions of the Royal Society of London, Vol. 181, 1890), in which the author combines the Kirchoff Method with Schwartz and Christofel conformal transforms and thereby solves some of the problems we solved. Having examined Michell's readings, we conclude that the method of generating and guidance grids is the most suitable for solving the problems under examination.

*Numbers in the margin indicate pagination in the foreign text.

$$vdv = gdx = gds\cos\vartheta = \frac{g\cos\vartheta d\varphi}{v},$$

where φ is the potential of the velocities. From this formula it follows that

$$v^3 = 3g\int \cos\vartheta d\varphi$$

or

$$\vartheta = \log\left(\frac{w}{v}\right) = -\frac{1}{3}\log\left(\frac{3g}{w^3}\int \cos\vartheta d\varphi\right), \tag{2}$$

where w is a certain constant velocity.

By ψ we symbolize an amount of flowing liquid and consider two imaginary values, $\varphi + \psi i$ and $\vartheta + \theta i$, according to which the generating and guidance grids are composed[3]. We assume that both these imaginary values are in fact functions of an argument variable $u = \xi + \eta i$, representing a point of a certain semi-plane (ξ is assigned any actual value, while η - only positive values), and we write:

$$\begin{aligned} \varphi + \psi i &= \chi(u), \\ \vartheta + \theta i &= \phi(u) + i\phi_1(u). \end{aligned} \tag{3}$$

Here, we select function $\chi(u)$ such that all its points of infinity lie on an axis ξ or in infinity, and such that all of axis ξ is symbolized by lines ψ = const. Functions $\phi(u)$ and $\phi_1(u)$ are selected such that they have only logarithmic points of infinity lying on axis ξ or in infinity, and such that all points of axis ξ satisfy either condition θ = const. or condition (2).

The first condition will be satisfied at those parts of the axis at which function $\phi(u)$ is actual, while function $\phi_1(u)$ is a purely imaginary value. We submit that on the remaining segments of axis ξ functions $\phi(u)$ and $\phi_1(u)$ will be active and we shall attempt to choose them such that they satisfy the second condition.

We make a substitution:

$$\phi_1(u) = \arccos(fu)$$

and determine $\phi(u)$ by the aid of $f(u)$ and $\chi(u)$ according to equation (2).

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[3]Modified Kirchhoff Method § 3.

We obtain:

$$\phi(u) = -\frac{1}{3}\log\left(\frac{3g}{w^3}\int f(u)\chi'(u)du\right),$$

so that the guidance grid of the unknown flow will be given by the formula: /91

$$\vartheta + \theta i = -\frac{1}{3}\, lg\left(\frac{3g}{w^3}\int f(u)\chi'(u)du\right) + i arccs[f(u)]. \qquad (4)$$

If in certain segments of axis ξ function $\phi_1(u)$ were actual, while function $\phi(u)$ consisted of the actual part and the imaginary part $\pm\pi$, then equation (2) would be satisfied because, in joining $\pm\pi$ to angle θ in formula (2), we thereby create a value under the logarithmic law from negative positive.

The entire success of solution now depends upon the good selection of functions $\chi(u)$ and $f(u)$.

§ 3. We assume that

$$f(u) = \frac{u}{c},\ \chi(u) = \frac{w^3}{2g}\,\frac{u^2}{c^2}. \qquad (5)$$

By formula (4) we find:

$$\vartheta + \theta i = -\log\left(\frac{u}{c}\right) + i\, arccs\left(\frac{u}{c}\right). \qquad (6)$$

From the second formula (5), we see that the generating grid in the supposition given here consists of equilateral hyperbolas provided by the equations:

$$\varphi = \frac{w^3}{2gc^2}\,(\xi^2 - \eta^2),\ \psi = \frac{w^3}{gc^2}\,\xi\eta. \qquad (7)$$

Streamline $\psi = 0$ in this grid is the upper half of the axis of the ordinate and the right and left halves of the axis of the abscissa. Moving along the hyperbolas ψ = const. in the direction of the diminishing η corresponds to a change in φ from $-\infty$ to $+\infty$.

Parameters ϑ and θ of the guidance grid seem to us to be differences in the parameters of a certain elliptic and polar grid, since, having assumed:

$$\xi = c e^{\vartheta_1} cs\theta_1, \quad \eta = c e^{\vartheta_1} sn\theta_1, \tag{8}$$

$$\xi = c\, cs\theta_2\, csh\vartheta_2, \quad \eta = c\, sn\theta_2\, snh\vartheta_2, \tag{9}$$

we find on the basis of formula (8) that

$$\theta = -\theta_1 + \theta_2, \quad \vartheta = -\vartheta_1 + \vartheta_2. \tag{10}$$

The grid expressed by these formulas is shown in Figure 1. It has a pole 0 at the initial point of the coordinates, and two foci, f and f'[4] at distances c from this initial point. All lines θ = const. proceed from pole 0 and once again intersect the axis of the abscissa at an oblique angle. In the case of a constant equal to zero the line of this family stretches to ∞ along axis 0η and, having branched into two branches at infinity, returns to foci f and f' along the segments of the axis of the abscissa ξf and ξ'f'. With a change in the constant from 0 to π/2, we obtain line θ = const., running to the right and intersecting the axis of the abscissa at segment 0f, while with a change in it from 0 to -π/2 we obtain lines running to the right and intersecting the axis of the abscissa at segment 0f'. /92

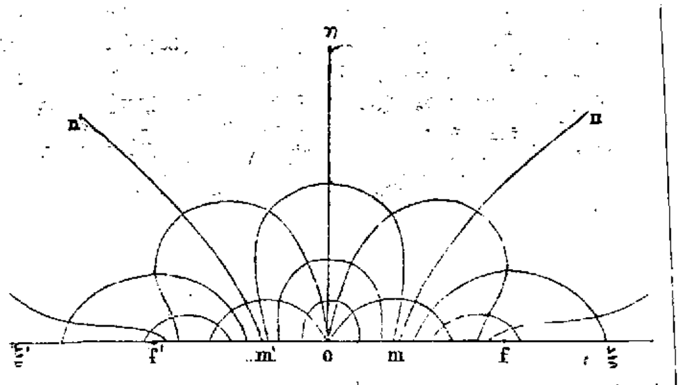

Figure 1.

The set of lines ϑ|= const. with const. = log2 produces two infinite lines mn and m'n', separating at infinity into two parts and encompassing the entire region under examination. With the change in the constant from log2 to ∞, lines ϑ|= const. take in pole 0 and diminish to the point of coincidence with it; with the change in the same constant from log2 to 0, lines ϑ|= const.

[4] We designate those points in which the streamline passes from the walls of the container to the edge of the jet.

intersect the axis of the abscissa to the right and to the left from focus f and to the right and to the left from focus f' and approach the lines which intersect the axis of the abscissa at points f and f' and which lie outside the region under examination.

We shall now superimpose the above-mentioned hyperbolic generating grid on the structure of the guidance grid such that the axes of the coordinates coincide. Since in the generating grid ψ changes from $+\infty$ to $-\infty$, we are dealing with an infinite stream of liquid. As a result of the fact that at all points of infinity $\vartheta = \log 2$ and $\theta = 0$, here the stream will flow in the direction of axis ox at a velocity of w/2. We find in our grids along the streamline that $\psi = 0$, changing φ from $-\infty$ to $+\infty$. We shall have the following on axis 0η: $\theta = 0$ and ϑ changes from log2 to ∞. At point 0, angle θ immediately changes from 0 to $\pi/2$ or to $-\pi/2$, depending upon whether we complete our passage to the segment of the axis of the abscissa of or of', while $\vartheta = \infty$ (i.e., velocity $v = 0$). At segments of and of' conditions (2) is satisfied, and angle θ changes from $\pm\pi/2$ to 0, and ϑ changes from ∞ to 0. At $f\xi$ and $f'\xi'$, angle $\vartheta = 0$ and ϑ changes from 0 to log2. /93

In reality, this corresponds to the flow of a stream of liquid (Figure 2) of streamline $\eta 0$, which at critical point 0 branches into two halves and forms the contours of a jet of liquid, of and of', passing in points f and f' to the immovable walls paralleling axis ox. Such a case could occur if we constructed a vertical canal $\xi ff'\xi'$ in the descending stream of a heavy liquid and joined it with the reservoir of air under a certain determined pressure. We will have results near those viewed as the ideal case of an unlimited flow if we construct an instrument as shown in Figure 2, and the dimensions of the vessel which we choose are significantly larger than the dimensions of the canal.

We shall determine the equation of contour of. On the basis of formula (7), (8), (9), and (10), we write for segment of:

$$\varphi = \frac{w^3}{2gc^2}\xi^2, \quad \vartheta = \log\frac{c}{\xi}, \quad \cos\theta = \frac{\xi}{c}. \tag{11}$$

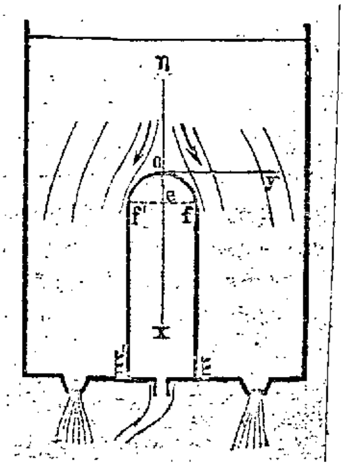

Figure 2.

On the other hand it is easy to see that for a viscous streamline there are differential equations:

$$\begin{aligned} dx = ds\, cs\vartheta = \frac{d\varphi}{v} cs\vartheta = \frac{e^{\vartheta}}{w} cs\vartheta d\varphi, \\ dy = \frac{e^{\vartheta}}{w} sn\theta d\varphi. \end{aligned} \qquad (12)$$

Expressing φ and ϑ by θ by the aid of formula (11) and substituting in formula (12), we obtain: /94

$$\begin{aligned} dx = -\frac{w^2}{g} cs\theta sn\vartheta d\theta, \\ dy = -\frac{w^2}{g} sn^2\theta d\vartheta. \end{aligned}$$

Integrating and determining the arbitrary constants under conditions $x = 0$, $y = 0$ with $\theta = \pi/2$, we obtain:

$$\begin{aligned} x = \frac{w^2}{2g} cs^2\vartheta, \\ y = \frac{w^2}{2g}\left(\frac{\pi}{2} - \vartheta + \frac{1}{2} sn 2\theta\right). \end{aligned} \qquad (13)$$

These formulas show that in Figure 2

$$eo = \frac{w^2}{2g},\ ef = \frac{w^2}{2g} \cdot \frac{\pi}{2},$$

and that the equation of contour of is:

$$y = \frac{w^2}{2g}\left(arcsn\sqrt{\frac{2gx}{w^2}} + \sqrt{\frac{2gx}{40^2}}\sqrt{1 - \frac{2gx}{w^2}}\right).$$

§ 4. We now move on to the hypothesis that on the jet contour liquid pressure and air pressure are mutually compensated for by the normal force arising from the capillary tension of the surface of the liquid.

If we let p symbolize the pressure within the liquid and p_1 air pressure lying along the contour of the jet, we will have:

$$\begin{aligned} p = const. - \frac{v^2\rho}{2}, \\ p_1 = p + \frac{\mu}{R}, \end{aligned} \qquad (14)$$

where ρ is the density of the liquid, μ the coefficient of capillarity and R the radius of curvature of the contour of the jet, considered positive in the direction away from the mass of liquid. From these two written formulas we obtain:

$$\frac{1}{R} = a + bv^2 , \tag{15}$$

where

$$a = \frac{p_1 - const.}{\mu}, \; b = \frac{\rho}{2\mu} . \tag{16}$$

But

$$\frac{1}{R} = \pm \frac{d\theta}{ds} = \pm \frac{v d\theta}{d\varphi},$$

therefore

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$$\frac{d\theta}{d\varphi} = \pm \left(\frac{a}{v} + bv\right)$$

or

$$\theta = \pm \int \left(\frac{a}{v} + bv\right) d\varphi. \tag{17}$$

In this formula with positive R, one should choose sign (+), if angle θ increases in the direction of the flow of liquid and sign (-), if angle θ diminishes in this direction; with negative R one should proceed the opposite. Having established that in the problems under examination the generating and guidance grids are expressed by the same formulas (3), as in the above discussed case, we apply the former conditions to functions $\phi(u)$ and $\phi_1(u)$; it is only required that in the segment of axis ξ, which corresponds to the actual value of these functions, that equation (17) be satisfied rather than equation (2). It is easy to see, that for this one must state the following:

$$\phi_1(u) = \pm \int \left(\frac{a}{w} e^{\phi(u)} + bwe^{-\phi(u)}\right) \chi'(u) du .$$

As a result of this, the guidance grid of the unknown flow will be given by the formula:

$$\vartheta + \theta i = \phi(u) \pm \int \left(\frac{a}{w} e^{\phi(u)} + bwe^{-\phi(u)}\right) \chi'(u) du . \tag{18}$$

§ 5. We assume that

$$\Phi(u) = -\frac{1}{2}\log\left(1 - \frac{u^2}{c^2}\right)$$

and write according to formula (18), accepting R to be positive and θ to be increasing

$$\Phi_1(u) = -\frac{bw}{c}\int \frac{u^2 - a^2}{\sqrt{c^2 - u^2}}\,\chi'(u)du,$$

where

$$a^2 = c^2\left(1 + \frac{a}{bw^2}\right). \tag{19}$$

Then we take the following value for $\chi'(u)$:

$$\chi'(u) = \frac{cdu}{bw(u^2 - a^2)},$$

thus it follows that

$$\chi(u) = \frac{c}{2baw}\, lg\,\frac{u - a}{u + a}. \tag{20}$$

The guidance grid of the problem under examination in the case of the assumptions made will be yielded by the formula:

$$\vartheta + \theta i = -\frac{1}{2}\log\left(1 - \frac{u^2}{c^2}\right) + i\arccos\left(\frac{u}{c}\right). \tag{21}$$

We shall now construct the generating and guidance grids which we have found. From formula (20) we see that the generating grid is a system of mutually orthogonal circles, one of which (Figure 3) passes through poles q and q' lying on axis ξ at distances α from the initial point of the coordinates and that it has a center in axis η; the others have their center in axis ξ, while points q and q' are mutually polar relative to them. For any point m of our semiplane, parameter ψ changes by a derivative of value /96

$$\frac{c}{2baw}$$

at angle qmq', while the value of parameter φ changes by a derivative of this value to a logarithm of the relationship of radii mq and mq'. With the change in ψ from 0 to

$$\frac{c\pi}{2baw}$$

line ψ = const. changes from segments of the axis of abscissa $q\xi$ and $q'\xi'$ to a segment of the axis of the abscissa of qq'; with the change in φ from $-\infty$ to $+\infty$, lines φ = const. change from an infinitely small circle encompassing pole q to an infinitely small circle encompassing pole q'.

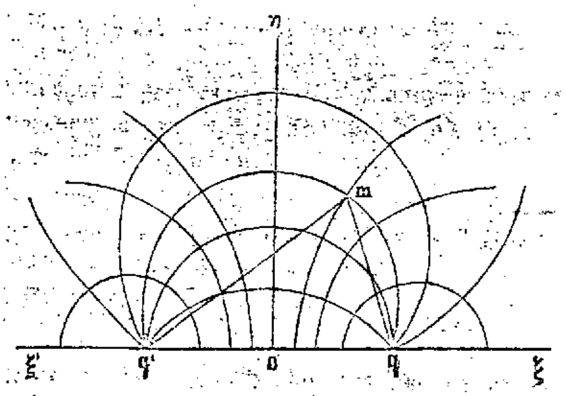

Figure 3.

The parameters of the guidance grid, provided by formula (21), provide us with the sum of parameters of the elliptical grid and the grid

$$\vartheta_1 + \theta_1 i = -\frac{1}{2}\log\frac{u^2 - c^2}{c^2} + \frac{1}{2}\pi i.$$

This last grid is shown in Figure (19) of our reading "The Modified Method of Kirchoff" and consists of Cassini ovals and a hyper- /97 bola passing through the poles at distances c from the initial point of the coordinates. We thereby obtain

$$\begin{aligned} \theta &= \theta_1 + \theta_2, \\ \vartheta &= \vartheta_1 + \vartheta_2, \end{aligned} \tag{22}$$

where θ_2 and ϑ_2 are determined according to formula (9), while θ_1 and ϑ_1 are determined according to the following formulas:

$$\begin{aligned} \theta_1 &= \frac{1}{2}\left(\pi - \arctan\frac{\eta}{\xi - c} - \arctan\frac{\eta}{\xi + c}\right); \\ \vartheta_1 &= -\frac{1}{4}\log\left(\frac{((\xi-c)^2+\eta^2)((\xi+c)^2+\eta^2)}{c^4}\right). \end{aligned} \tag{23}$$

The result of this presentation provides us with a grid shown in Figure 4. Points f and f', distant from the initial points of the coordinates by a distance c, serve as poles and foci for this grid. The lines of θ = const. pass from pole f or f' and intersect once again the axis of the abscissa at segment ff' at an oblique angle. With the change in the constant from 0 to $\pi/2$, these lines pass from focus f and change from an infinitely small line to a line running along a segment of the axis of the abscissa of $f\xi$ to infinity and returning along the axis of the ordinate $\eta 0$; with the change in the constant from $\pi/2$ to π, we obtain lines running from the pole f' and changing from a line running along segment f'ξ' of the axis of the abscissa to infinity and returning along the axis of ordinate 0η, to a certain infinitely small line. The lines of ϑ = const. with const. = log2 produce two infinite lines mn and m'n', branching into infinity at two segments and encompassing the entire region under examination. With the change in the constant from log2 to 0, the lines of the set under examination, ending between mn and m'n', change to lines which /98 intersect the axis of the abscissa in point 0 and which lie within the region under examination; with the change in the constant from log2 to infinity, these lines encompass poles f and f_1 and diminish to infinitely small dimensions. From the above-stated it follows that in all points of infinity

$$\theta = \frac{\pi}{2},\ \vartheta = \log 2.$$

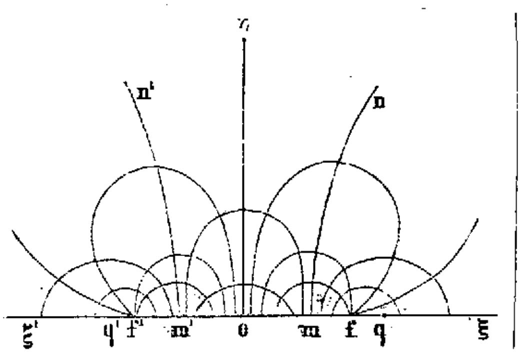

Figure 4.

We shall now assume that a is a positive value, i.e., $\alpha > c$ (according to formula 19), and we shall superimpose the generating grid of Figure 3 on the guidance grid of Figure 4, such that the axes of the coordinates coincide. Moving along line $\psi = 0$ in the direction of increasing φ, we will have the following: at segment qξ, angle $\theta = \pi/3$ and ϑ changes from value

$$\vartheta_0 = \log\left(1 + \frac{\alpha}{\sqrt{\alpha^2 - c^2}}\right)$$

to log2; in segment ξ'q', angle θ equals $\pi/2$, while ϑ changes from log2 to ϑ_0. This corresponds to Figure 5 in an actual flow of liquid of a straight streamline qξq', running parallel from y. We now move along line

$$\psi = \frac{c\pi}{2baw} \tag{24}$$

in the direction of increasing φ. We will have angle $\theta = \pi/2$ in a segment of axis of abscissa qf, while ϑ changes from ϑ_0 to ∞; in point f, angle θ

immediately changes to 0 and $\vartheta = \infty$; at the segment of axis of abscissa ff', with condition (17) satisfied, angle θ changes from 0 to π while ϑ changes from ∞, decreasing to 0 (at point 0) and then again increases to ∞; at point f' angle θ immediately changes by π/2 and θ = ∞; finally, at segment f'q', angle θ = π/2 and ϑ changes from ∞ to ϑ_0. This corresponds in an actual flow of liquid to straight streamline qf, and jet contour fof', in which capillary tension is active, and straight streamline fq'. The flow of liquid examined by us will be included between the two found contours[5]. Here our assumption /99 that R is positive and θ increases will be satisfied. If we add a symmetrical flow to the found flow, on the other side of axis y, we will obtain the solution to the problem concerning the form of the air bubble within the liquid flowing in canal qq'rr'; if we add to the entire system velocity

$$v_0 = w:\left(1 + \frac{\alpha}{\sqrt{\alpha^2 - c^2}}\right) \tag{25}$$

in a direction counter to axis o'y, then we obtain the solution to the problem of movement of the air bubble within the liquid running in the canal.

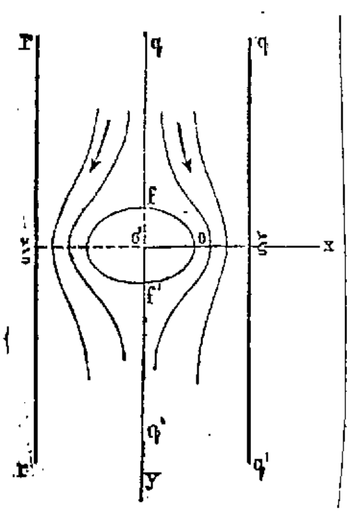

Figure 5.

We shall construct the equations of contour fof'. According to formulas (22), (9), (23) and (20), we have at segment of:

$$\theta = \theta_2,\ \xi = c.cs\theta,\ \vartheta = -\frac{1}{2}\log\left(1 - \frac{\xi^2}{c^2}\right) = -\log sn\theta,$$
$$\varphi = \frac{c}{2b\alpha w}\log\frac{\alpha - \xi}{\alpha + \xi} = \frac{c}{2b\alpha w}\log\frac{\alpha - c.cs\vartheta}{\alpha + c.cs\theta} \tag{26}$$

We substitute values determined from these formulas in equation (12):

$$dx = \frac{c^2}{bw^2}\,\frac{cs\theta d\theta}{\alpha^2 - c^2 cs^2\theta},$$
$$dy = \frac{c^2}{bw^2}\,\frac{sn\theta d\theta}{\alpha^2 - c^2 cs^2\theta}.$$

We integrate and determine the arbitrary constant under the condition that with θ = 0 we have x = 0, while with θ = π/2 we have y = 0. We obtain:

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[5]Modified method of Kirchoff, § 2. See portion on direction in which [Translator's Note: symbol blurred] increases.

$$x = \frac{1}{bw^2}\frac{c}{\sqrt{a^2-c^2}}\arctan\frac{c.sn\theta}{\sqrt{a^2-c^2}},$$
$$y = \frac{1}{bw^2}\frac{c}{2\alpha}\log\frac{\alpha - c.cs\theta}{\alpha + c.cs\theta} \qquad (27)$$

where for obtaining the points of contour fof' we must assign θ values from 0 to π. We see that the semiaxis of the bubble will have a length:

$$oo' = \frac{1}{bw^2}\frac{c}{\sqrt{a^2-c^2}}\arctan\frac{c}{\sqrt{a^2-c^2}},$$
$$o'f' = \frac{1}{bw^2}\frac{c}{2\alpha}\lg\frac{\alpha+c}{\alpha-c}.$$

It is easy to see that 00' > 0'f'. This follows from the obvious inequality

$$\int_0^c \frac{dx}{\sqrt{(a^2-c^2)(a^2-x^2)}} > \int_0^c \frac{dx}{a^2-x^2},$$

which, on accomplishing integration, yields:

$$\frac{1}{\sqrt{a^2-c^2}}\,arcsn\,\frac{c}{\alpha} > \frac{1}{2\alpha}\lg\frac{\alpha+c}{\alpha-c}.$$

With respect to the width of the canal, it is found from formulas (24) and (25) by dividing the amount of flowing liquid by the velocity in the infinitely long portions of the canal:

$$\xi\xi = \frac{2\psi}{v_0} = \frac{\pi}{bw^2}\frac{c}{\alpha}\left(1+\frac{\alpha}{\sqrt{a^2-c^2}}\right). \qquad (28)$$

Having decided on relationship c/α and value w, we determine according to formulas (28) and (25) the width of the canal and velocity v_0, while according to formula (27) we find the shape of the air bubble.

§ 5. We are limited in this note by two cited examples, but we feel that formulas (4) and (18) with appropriate selection of the arbitrary functions entering them still lead to solution of many other interesting problems concerning the movement of a liquid under the effect of the force of gravity and under the effect of capillary tension in a free contour jet.

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